

Improving Forecast Accuracy by a Segmented Rate of Change in Technology Forecasting Using Data Envelopment Analysis (TFDEA)

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Abstract--Technology forecasting using data envelopment analysis (TFDEA) captures technological advancement from the evolution of the state-of-the-art (SOA) frontier. Within this process, TFDEA combines rates of changes (RoC) from past technologies that have been superseded by superior technologies. However, it was occasionally observed in previous applications that forecasting based on a single aggregated RoC did not consider the unique growth patterns of each technology segment, which resulted in a conservative or aggressive forecasting. This study proposes a procedure to improve the forecasting accuracy by identifying local rates of change for each frontier segment that may represent different product families. This approach is applied to six previously published applications using a rolling origin hold-out sample tests to validate its performance compared to the traditional TFDEA approach. The results indicate that the segmented rate of change approach determines different rates of change for product niches that result in more accurate forecasts.

I. INTRODUCTION

As technology becomes sophisticated, there are few technologies that possess only a single technical capability. This raises a fundamental question about the technology forecasting problem: what is the best way to combine growth patterns of each attribute to describe the multi-objective technology system? To tackle this multi-attribute problem, modern technological forecasting studies frequently use frontier analysis methods. The idea is to form a surface that can represent the same level of technology systems at given point in time. The evolution of surfaces is then monitored to capture the rate of change (RoC) by which future technological possibilities can be estimated. In particular, the non-parametric frontier approach forms the technology frontier without a predefined function to generate the tradeoff surface. This property allows the model to reflect the distinct characteristics of the application area by adapting to empirical data instead of relying on arbitrary assumptions [1].

Technology forecasting using data envelopment analysis (TFDEA), which is a non-parametric frontier approach, iterates a frontier formation process over time to capture the rate of technological progress [2]. Its unique characteristic of utilizing extreme data, i.e., state-of-the-art (SOA) technologies, has provided accurate forecasting as well as managerial implications in a wide range of applications since the first introduction in PICMET '01 [3]–[9]. Furthermore, as TFDEA has drawn widespread attention recently, studies focusing on extension of the methodology such as decision making unit (DMU) filtering [10], time variable rate of change [11], and residual diagnostics [12] are also actively conducted.

However, it was occasionally observed in previous applications that forecasting based on a single aggregated RoC did not consider the unique growth patterns of each technology segment, which resulted in a conservative or aggressive forecasting. This issue, in particular, becomes more visible when the application area contains distinct progress patterns identified from multiple technology segments.

This study addresses this issue and proposes a procedure for segmented RoC based on benchmarking references obtained from the DEA model. This is organized as follows. In the next section, Section 2, the notion of segmented RoC is illustrated to supply insight into the problem being discussed. In Section 3, TFDEA formulation incorporating the proposed procedure is introduced. In Section 4, our approach is tested for preceding applications to validate its performance, and relevant issues are discussed. Finally, in Section 5 we summarize our results and suggest possible future research directions.

II. SEGMENTED RATE OF CHANGE

Since TFDEA has at its core the widely used technique of DEA, TFDEA inherits the ability to provide many of the rich results. One of the key results yielded by DEA is the identification of targets and efficient peers [13]. Specifically, DEA constitutes the frontier of a production possibility set (PPS) based on “best practice” DMUs. Within this framework, relative efficiency is determined by comparing the performance of each unit against that of a (virtual) target formed by efficient peers. A practical interpretation is that efficient peers can serve as role models which inefficient DMUs can emulate so that they may improve their performances. In other words, those benchmarks have a mix of input-output levels similar to that of DMUs being compared, which indicates that they are likely to operate in analogous environments and/or to favor similar operating practices [14].

The implementation of TFDEA relies on a series of benchmarking processes over time [2]. This is depicted in Fig. 1, assuming an output-oriented DEA model under variable returns to scale (VRS) [15]. The frontier year is the point in time at which the analysis is conducted. Products G, H, and I are identified to be the most competitive and therefore define the SOA frontier. Products A~F, in contrast, have been superseded by superior products and hence are located below the frontier. Products J and K are future products, i.e., sets of specifications used as forecasting targets that are placed beyond the current SOA frontier.

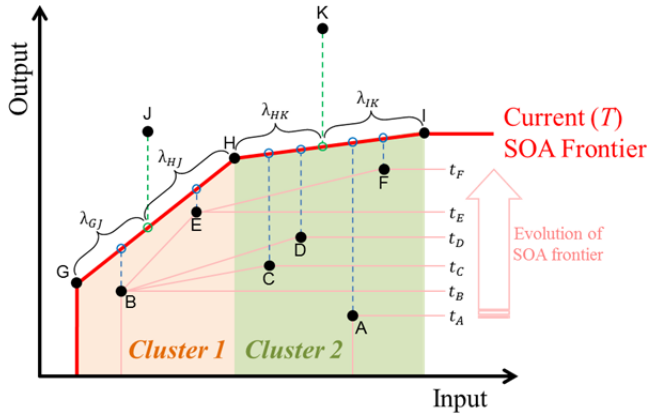


Figure 1 Evolution of the SOA frontier

The TFDEA process can be understood as three procedural stages. First, it iterates the DEA process to obtain efficiency scores of products both at the time of release and at the frontier year. Second, it estimates a RoC that represents how fast products have been replaced by the next generation products. In other words, the RoC indicates a potential growth rate of the SOA frontier in the future. Finally, the model makes a forecast of future products based on the average RoC.

One may notice that the original TFDEA process simply aggregates RoCs from the past products and uses the average RoC to make a projection without taking product segmentation into account. However, as previously discussed, DEA provides pragmatic information regarding benchmarks, which enables an identification of distinct product clusters [16]. This information can be obtained either by reference sets in the envelopment model or by weighting schemes in the multiplier model.

For example, two different clusters are identified in Fig. 1. The first cluster can be characterized by an optimized weighting scheme, that is, a facet connecting products G and H. This can be interpreted that inefficient products pertinent to this cluster, namely B and E, may have similar mixes of input-output levels such that a corresponding weighting scheme will show them in the best possible light. This can also be recognized as a reference set in the envelopment model since their performances are compared against virtual targets constituted by efficient peers, namely products G and H.

In the same manner, the second cluster can represent another weighting scheme, that is, a facet connecting products H and I. Even though the underlying products, A, C, D, and F, have less efficient absolute levels, they must have similar ratios of the input-output levels that require the common weighting scheme to optimize their operations [14]. The envelopment model, likewise, will constitute virtual products interpolated by products H and I for these inefficient products.

The idea of segmented RoC arises when there is a need to draw a distinction between each cluster; hence, the growth

potential should be explained by local RoCs rather than a universal RoC. In our example, one may notice that cluster 2 has observed faster RoCs than cluster 1. Specifically, products B and E did not show a large performance gap compared to the current SOA frontier even though the old product B, in particular, had stayed on the SOA frontier for a long time and only recently became superseded. This implies that the technological progress within cluster 1 has been neither fast nor frequent. In contrast, products pertinent to cluster 2 have shown successive replacements with substantial performance advancement over time. This may imply that more engineering effort has been invested in cluster 2-type products, which results in more frequent introductions of advanced products over time.

Once distinguishing clusters are identified with corresponding RoCs, it is readily possible to make a forecast using those local RoCs. For example, the estimated arrival of future product J can be determined by measuring how far it is from the current SOA frontier, i.e. super-efficiency, and then extracting the root of that distance using local RoCs from cluster 1 given the fact that it is projected to the frontier facet of cluster 1. In the same manner, the arrival of future product K can be estimated using local RoCs from cluster 2. One may predict that if both products were achievable with the same amount of performance improvement, the arrival of product K might be earlier than that of product J since faster progress is expected from cluster 2-type products. In other words, requiring the same amount of time to reach the technological level of product J would entail significant development risk.

III. TFDEA FORMULATION

We now turn to the TFDEA formulation incorporating the proposed approach.

The first stage, shown by (1)-(7), performs efficiency measurement in a time series manner so that the evolution of the SOA frontier can be monitored. Specifically, x_{ij} represents the i th input and y_{rj} represents the r th output for each technology $j = 1 \dots n$, and $j = k$ identifies the technology to be evaluated. The variable $\phi_k^{h \in \{R, C\}}$ represents the radial output efficiency of technology k at the time of release (R) and current frontier time (C) in which the forecast is conducted. That is, ϕ_k^R measures the amount by which technology k is surpassed by the technologies available at the time of release since constraint (4) allows the reference set of technology k to only include technologies that had been released up to t_k . Similarly, ϕ_k^C can be interpreted as how superior technology k is against the current SOA frontier by constraint (5). Note that the “current time” is defined as a fixed time T , which can be either the most recent time in the dataset or a certain point in time as a forecasting origin when the time series hold-out sampling is performed. The variable, λ_{jk}^h , describes how much of technology j is used in setting a target of performance for technology k .

$$\max \sum_{k=1}^n \left[\phi_k^h - \varepsilon \left(\frac{\sum_{j=1}^n \lambda_{jk}^h \cdot t_j}{\sum_{j=1}^n \lambda_{jk}^h} \right) \right] \quad (1)$$

$$s. t. \sum_{j=1}^n \lambda_{jk}^h \cdot y_{rj} \geq \phi_k^h \cdot y_{rk}, \quad r = 1, \dots, s \quad (2)$$

$$\sum_{j=1}^n \lambda_{jk}^h \cdot x_{ij} \leq x_{ik}, \quad i = 1, \dots, m \quad (3)$$

$$\lambda_{jk}^R = 0, \quad \forall (j, k) | t_j > t_k \quad (4)$$

$$\lambda_{jk}^C = 0, \quad \forall (j, k) | t_j > T \quad (5)$$

$$\sum_{j=1}^k \lambda_{jk}^h = 1, \quad \forall k \quad (6)$$

$$\lambda_{jk}^h \geq 0, \quad \forall j, k, h \in \{R, C\} \quad (7)$$

The objective function (1) also incorporates minimizing effective dates. This allows the model to ensure reproducible outcomes from possible alternate optimal solutions [17]. Note that in the case of the VRS model, constraint (6) would allow replacing the denominator in the second term with a constant 1, making the objective function, (1), linear. Here, it is imperative that the value of a non-Archimedean infinitesimal, ε , not be implemented as a finite approximation to avoid inaccuracies and erroneous results [18]. Instead, the actual implementation is to use a two-stage preemptive linear programming to first identify the radial efficiency and then to maximize (or minimize) effective dates according to the need.

The second stage calculates the RoC, γ_k^C , by taking all technologies that were efficient at the time of release, $\phi_k^{R*} = 1$, but were superseded by new technologies at the current frontier time C , $\phi_k^{C*} > 1$. Having calculated RoCs of past technologies in (8), the idea of segmented RoC can then be implemented by taking the weighted average of RoC for each technology on the current SOA frontier. This leads to the calculation of local RoCs in (9), where δ_j^C represents the local RoC driven by technology j at current time T . Note that technology j has an efficiency score of 1 at the current frontier; in other words, it is one of the SOAs that constitutes the frontier onto which future technologies are to be projected. The numerator of (9) indicates the weighted sum of RoCs from past technologies that have set technology j as a (or one of) benchmark(s). The denominator indicates the accumulated contribution of technology j to the evolution of the SOA frontier. Consequently, δ_j^C represents the local RoC that only counts RoCs in which SOA technology j has been used as a benchmark.

$$\gamma_k^C = \frac{1}{\sum_{j=1}^n \lambda_{jk}^{C*} \cdot t_j} \cdot t_k, \quad \forall k | \phi_k^{R*} = 1, \phi_k^{C*} > 1 \quad (8)$$

$$\delta_j^C = \frac{\sum_{k=1}^n \lambda_{jk}^{C*} \cdot \gamma_k^C}{\sum_{k=1, \gamma_k^C > 0} \lambda_{jk}^{C*}}, \quad \forall j | \phi_k^{C*} = 1 \quad (9)$$

The last stage makes a forecast of the arrival of future technologies. In (10), $t_k^{forecast}$ represents the estimated time of arrival of future technology k ; therefore, ϕ_k^{C*} indicates the

super-efficiency of technology k forecasted against the current SOA frontier. Since future technologies, namely target sets of specifications, are to be located beyond the current SOA frontier in the PPS, the reciprocal of ϕ_k^{C*} reflects the largest proportion that any one of its output levels is of the maximum level that output observed from current SOA could take given input levels. The variable $\lambda_{j,k}^{C*}$ can be interpreted as an indicator for the classification of future technology k defined by current SOA technologies. Therefore, the individualized RoC for the future technology k can be calculated by combining the local RoC of SOA technology j , δ_j^C , that constitutes the frontier facet onto which technology k is being projected. Note that traditional TFDEA uses a constant (average) RoC to project all future technologies. Finally, the forecasted time $t_k^{forecast}$ is obtained by the sum of estimated elapsed time and the effective date. For a more detailed review of the TFDEA process, the interested reader is referred to earlier studies [2], [7], [19].

$$t_k^{forecast} = \frac{\ln\left(\frac{1}{\phi_k^{C*}}\right)}{\ln\left(\frac{\sum_{j=1}^n \lambda_{j,k}^{C*} \cdot \delta_j^C}{\sum_{j=1}^n \lambda_{j,k}^{C*}}\right)} + \frac{\sum_{j=1}^n \lambda_{j,k}^{C*} \cdot t_j}{\sum_{j=1}^n \lambda_{j,k}^{C*}}, \quad \forall k | t_k > T \quad (10)$$

IV. VALIDATION AND DISCUSSION

We now relate our approach to preceding works to show its forecasting performance. Fundamentally, the true accuracy of forecasting model is determined by the future events that were not known in model building process. However, this so-called 'real time assessment' has practical limitations, which makes a holdout sample test that measures how the model is able to reproduce data already known but not used in construction of the model commonplace in forecasting literature [20]. The resulting forecast deviations, i.e. difference between estimated data and reserved data, can therefore provide an accuracy measure (or the goodness of fit) of the forecast model being considered. This is also useful to compare the performance of different models on the same data [21].

To validate the performance of proposed approach, we conducted holdout sample tests using both constant RoC and segmented RoC on six datasets. Note that a rolling origin was implemented to obtain a sufficient number of forecasts as well as to desensitize the error measures to special events at any single origin [22]. It should be also noted here that we adopted the accuracy measure of root mean square error (RMSE) to represent forecasting errors since our forecast is the arrival of technologies, i.e. single scale with non-zero occurrence, estimated from their performance levels [23]. In addition, deviation distributions were tested to distinguish their differences from random variations with statistical significance. Table 1 summarizes comparative results of forecasting accuracies.

TABLE 1 FORECAST ACCURACY COMPARISONS

Application area	RMSE (Root mean square error)		Deviation statistics (95% confidence interval)		Paired <i>t</i> test	
	Constant RoC	Segmented RoC	Constant RoC	Segmented RoC	<i>t</i> -stat	<i>p</i> -value
Commercial airplane [4]	11.9208	6.3084	-9.06(±5.18)	-3.56(±3.65)	-4.3653	0.0023
Fighter jet [19]	7.8229	7.2524	-7.22(±3.38)	-6.32(±3.17)	-2.1274	0.0454
Battle tank [10]	23.1312	16.7987	-15.57(±7.62)	-9.30(±6.30)	-5.3973	0.0001
Liquid crystal display (LCD) [6]	2.3061	2.1508	+0.63(±0.27)	+0.35(±0.30)	6.7182	0.0000
Hybrid electric vehicle [24]	3.4176	3.3329	-2.33(±1.70)	-2.26(±1.67)	-3.2221	0.0105
Digital single lens reflex (DSLR) [25]	2.6333	2.6271	-0.43(±0.36)	-0.15(±0.33)	-3.8553	0.0002

In all cases, segmented RoC showed not only smaller forecasting errors, i.e. $RMSE_{Constant\ RoC} > RMSE_{Segmented\ RoC}$, but also statistically distinct distributions closer to zero than that of constant RoC, i.e. $\mu_{Constant\ RoC}^{Deviation} > \mu_{Segmented\ RoC}^{Deviation}$ ($p < 0.05$). One may infer that forecasting accuracy improvement would be more significant if unique segments were identified with a great local RoC contrast to one another and future technologies were subject to those unique segments. This can be shown by comparing the constant RoC with individualized RoCs. Figure 2 contains this information. Note that RoCs were normalized to show their distribution in comparison to the constant namely average RoC that was set to be 100%. It is seen that in case of commercial airplane and battle tank, individualized RoCs for forecasting targets show skewed distributions from constant RoC. That is, most of forecasting targets were subject to relatively fast progressing segments that a constant RoC yielded extremely conservative forecasts whereas the segmented RoC approach could reflect those

variations, which resulted in considerable accuracy improvements.

On the contrary, when the local RoC of a certain segment by which most future technologies are classified was close to the constant RoC, the impact of segmented RoC would be marginal even if a wide range of local RoCs was identified. This can be seen from the case of DSLR application in which a constant RoC could reasonably represent the variations of individualized RoCs as an average value.

A special case can occur when the regions or clusters do not contain past products that have been surpassed. In this case, a product may not have a local RoC. Graphically, this would occur in Fig. 1 if products B and E were not included, which would then result in G failing to have a local RoC. In place of the G's local RoC is then assumed to be the average RoC of all SOA products (H and I). Another approach would be to average the RoC for products that are on the same facet(s) of the efficiency frontier as G (simply H).

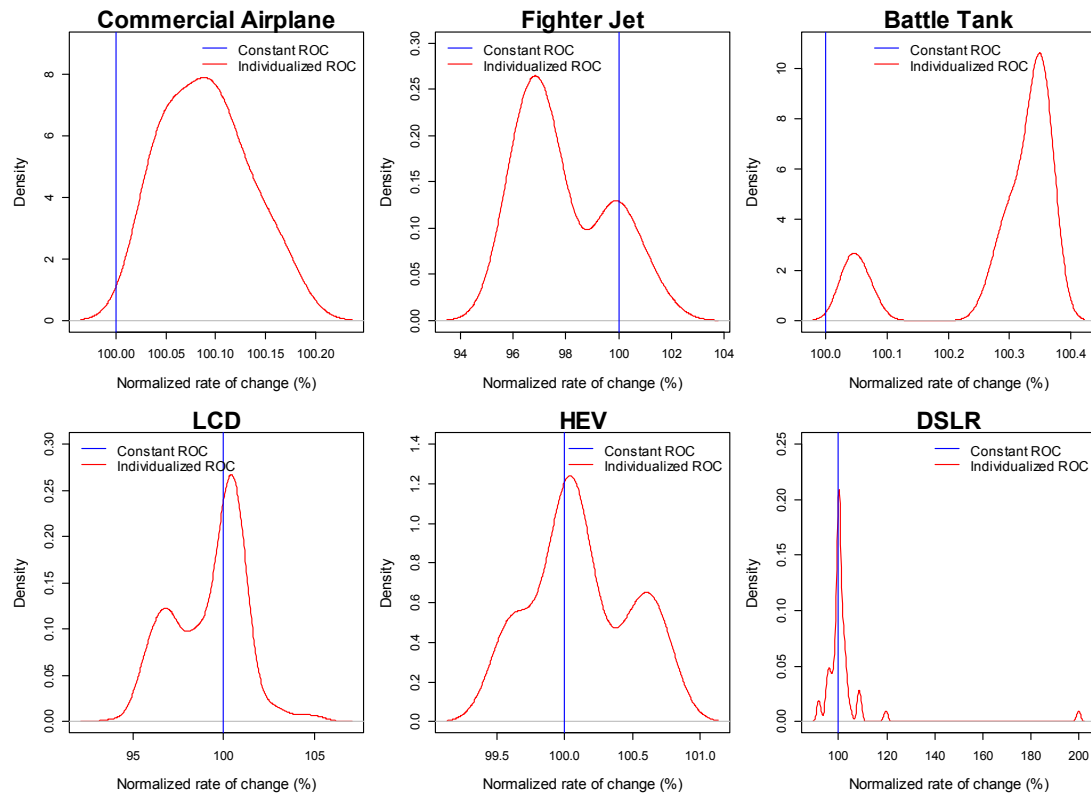


Figure 2 Relative comparison of segmented RoC with constant RoC

This paper provided the output-oriented formulation with VRS. The input-oriented formulation is a simple variation. Returns to scale other than VRS do not have a simple linear secondary objective function for resolving multiple optima and is then solved as a linear approximation described in [26].

VI. CONCLUSION

In this paper, we have proposed a procedure that is intended to utilize a segmented RoC within the TFDEA framework. Constant RoC was traditionally used as a single indicator to represent the momentum of technological progress without considering that there may be a different RoC for each technology segment. Empirical illustrations have shown that the proposed approach can capture local RoCs, and employing individualized RoCs to make a forecast improves the forecast's accuracy.

Obviously, there might be alternate ways to identify distinct advancement patterns in TFDEA. One of which could consider non-radial target setting approaches. The traditional DEA model is labeled as "radial" since it gives preemptive priority either to conservation of the input or to expansion of the output depending on model orientation. This implies that radial approach may not capture the technological advancement within structural characteristics or functional improvements while the technology systems' objective might often be the desire to change the mix of them. Non-radial target setting approaches that allow the identification of closest targets, axiomatic targets, restricted targets, and scale-efficient targets could therefore set more realistic targets whereby diverse patterns of technological advancement can be explored.

Another direction of future work could also investigate the varying RoCs over time. The local RoCs can provide information about the number of distinct segments within which differing rates of technological advancement have been captured. One can utilize this information as an indicator of market dynamics such that identification of product niche or disruptive potential from fast growing segments.

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